Leibniz Universität Hannover
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Exam
on
Manufacturing Systems Modeling and Analysis
Winter Term 2019-2020

Hints:

1. The exam consists of **9** pages (including this front page). Please check that your copy is complete and complain immediately if it is not.
2. Answer all questions and solve all given problems.
3. You are given 60 minutes to work on the exam and you can score a total of 60 points.
4. You may answer the questions using either the German or the English language.
5. **Show your work!** If you use a formula to solve a problem, present it in its general form first!
6. You may use a single double-sided and hand-written help sheet in letter format or DIN-A4 format with any content you may find helpful to work on the exam.
7. You may use a pocket calculator.
8. For this exam written in a university computer lab you may use the desktop computer provided by the university and the Matlab files related to this exam that have been provided in class and stored locally on this machine for your usage while working on the exam.

Personal data:

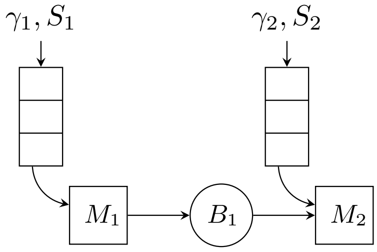
Family name	Given name	Matriculation number	Study program

Rating:

Task	1	2	3	Sum
Score				

1. **Analysis of a Markovian two-stage flow line with unreliable machines, limited buffer capacity between the machines, and limited storage of auxiliary material at the stations (30 P.)**

Consider a two-stage flow line with limited buffer capacity C between machines M_1 and M_2 as shown below.



The first machine is never starved and the last machine is never blocked with respect to the actual workpieces processed by the two machines. However, at both machines a unit of auxiliary material is required each time the respective operation is performed. This material is supplied to the machines by vehicles which arrive with exponentially distributed inter-arrival times with rates $\gamma_i, i = 1, 2$. If the vehicle arrives, it fills the local storage for the auxiliary material up to a level $S_i, i = 1, 2$.

Furthermore, assume that the machines are unreliable and can fail while they are operating. Times to failure as well as repair times are exponentially distributed with rates $p_i, i = 1, 2$ and $r_i, i = 1, 2$, respectively. Processing times are exponentially distributed with rates $\mu_i, i = 1, 2$ at the two machines. We assume blocking after service (of the first machine).

The state of the system at any moment in time can be described as a vector $(k_u, k_d, n, \alpha_1, \alpha_2)$ where k_u denotes the number of units of auxiliary material at the first station and likewise k_d the respective number at the last station. The number of workpieces in the system that have already been processed by the first station is n. Let $\alpha_i = 1$ denote the state of machine i being up (and $\alpha_i = 0$ the state of machine i being down).

Assume that a mapping function $StN(k_u, k_d, n, \alpha_1, \alpha_2)$ maps the states onto integer state numbers to be used in setting up the generator matrix Q of a continuous time Markov chain (CTMC).

a) Using pseudo code or Matlab code, construct the required nested loops to build up the complete generator matrix Q of the CTMC model of the system, taking care of all the possible events. (21 P.)

b) Assume that you have been able to determine all the steady-state probabilities $\text{Prob}(k_u, k_d, n, \alpha_1, \alpha_2)$, give the equations to compute

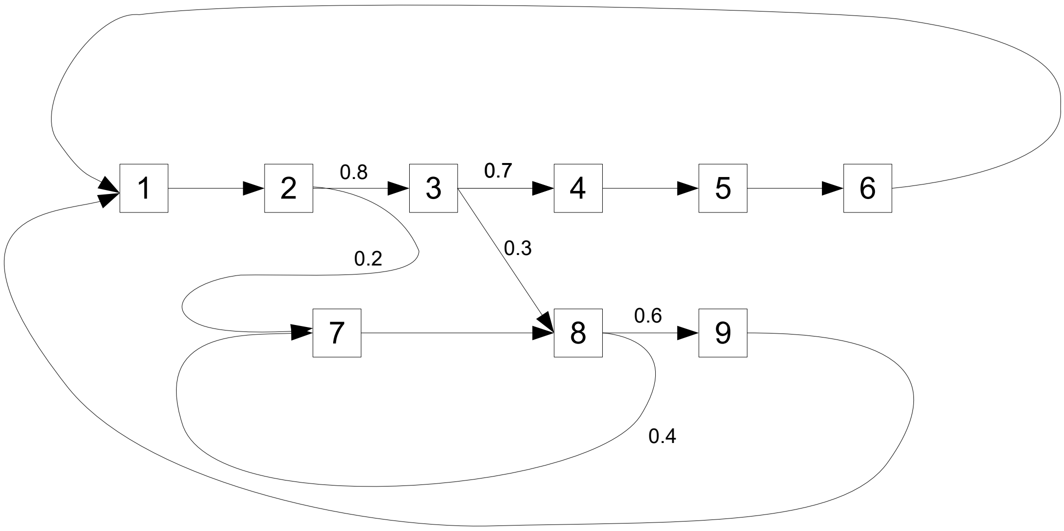
i. the production rate PR_1 via the first machine, (3 P.)

ii. the average inventory $\bar{n}_1$ of main work pieces already processed by machine M_1, but not by machine M_2, and (3 P.)

iii. the average inventory $\bar{k}_d$ of auxiliary material at the second machine. (3 P.)

2. **Analysis of a closed single-server single-product network** **(15 P.)**

a) Analyze using an appropriate Matlab program the network of single-server work stations used to produce a single product type that is depicted in the following figure.



The product units are mounted on pallets as they enter the system at station 1. Finished products are dismounted from the pallet behind station 6. The throughput of station 6 is hence equal to the throughput of the system in terms of completed product units. Transportation times as well as the times required for the mounting and dismounting processes are negligible.

The routing probabilities of the stations performing a Markovian splitting of the departure stream are given in the figure. Assume that the processing times follow an exponential distribution. The following table gives the expected processing times $\mathrm{E}[T_s(i)]$ per station i:

Station i	$\mathrm{E}[T_s(i)]$ [min]
1	15
2	3
3	30
4	25
5	10
6	20
7	40
8	5
9	18

Assuming a constant number of $w = 10$ work pieces circulating in the system, determine

i. the throughput of the system, (1 P.)

ii. the work in process, cycle time, throughput and server utilization at each of the stations, and (4 P.)

iii. determine the bottleneck of the system: (1 P.)

iv. What happens to throughput and cycle time of the system as the number of work-pieces circulating in the system increases further? Why? (3 P.)

b) Explain and justify

   i. the fundamental equation on which the Mean-Value-Analysis algorithm is based (2 P.)

   ii. as well as the conditions under which it yields exact results and (2 P.)

   iii. why it leads to an iterative algorithm to determine performance measures. (2 P.)

3. **Batch moves** **(15 P.)**

Consider a situation in which (work) station i sends jobs directly to station j. The jobs are transported from station i to station j by batch moves of size k. Jobs arrive at station i with expected inter-arrival times $\mathrm{E}[T_a(i)]$ and depart from this station with a squared coefficient of variation $c_d^2(i)$. At the receiving station j the expected value of the service time is $\mathrm{E}[T_s(j)]$ and its squared coefficient of variation is $c_s^2(j)$.

Derive and explain a formula to approximate the expected cycle time $CT(j)$ per job at station j and explain the effect of an increase of the batch size k on the cycle time, the inventory, and the throughput!