Leibniz Universität Hannover
Wirtschaftswissenschaftliche Fakultät
Institut für Produktionswirtschaft
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Exam on
Manufacturing Systems Modeling and Analysis
Winter Term 2020-2021

Step 1: Enter your data below:

First name:

Last name:

Course of study:

Matriculation number:

Date:

Starting time and ending time of the exam:

Step 2: Read and sign the following declaration:

I am aware that attempted cheating will result in the grade "failed" according to the examination regulations. I am enrolled as a regular or exchange student at Leibniz Universität Hannover for the current semester and am not on leave of absence. I expressly declare myself fit to take the examination and agree to the conditions of the examination and the examination procedure. I have taken note of the general instructions. **I confirm that I have completed the online exam independently without the help of others and without helping other exam candidates.** The handwritten text in this file shows my own work.

Place:

Date:

Signature:

Rating:

Task	1	2	3	4	Bonus points	Sum
Score						

Hints:

1. The exam consists of **11** pages (including the front page and this page with hints). Please check that your file is complete.
2. Answer all questions and solve all given problems.
3. You are given 60 minutes to work on the exam and you can score a total of 60 points. You are given additional 30 minutes for downloading, printing, scanning, and uploading your results. The results have to be uploaded by 10:45 am, February 2, 2021.
4. You may answer the questions using either the German or the English language.
5. **Show your work!** If you use a formula to solve a problem, present it in its general form first!
6. You may use any source, e.g., textbooks, notes taken during the course, files that have been given to you, but you have to work on your own, not receiving or providing help from or to anybody during the time span of the exam.
7. Name your single PDF file which you upload as follows:
   "`ManSys_WS2021_<MatriculationNumber>_<FirstName>_<LastName>.pdf`"

1. **The exponential distribution** **(18 P.)**

   Assume that service times T at a machine are exponentially distributed with service rate $\mu = \frac{1}{2}\mathrm{h}^{-1}$.

   a) Give for this random variable T both general formulas and the concrete numerical values (including appropriate units) of

      i. the expected value, (2 P.)

      ii. the standard deviation, (2 P.)

      iii. and the coefficient of variation. (2 P.)

   b) Determine the probability $\mathrm{Prob}(1\ \mathrm{h} \leq T \leq 4\ \mathrm{h})$! (3 P.)

   c) Determine the conditional probability $\mathrm{Prob}(T \geq 4\ \mathrm{h} | T \geq 1\ \mathrm{h})$! (3 P.)

d) Determine the conditional probability $\mathrm{Prob}(T \geq 1\ \mathrm{h} | T \geq 4\ \mathrm{h})$! (3 P.)

e) Explain what is meant when we say that the exponential distribution possesses the "memorylessness property" and explain why this is relevant for the analysis of stochastic processes and systems! (3 P.)

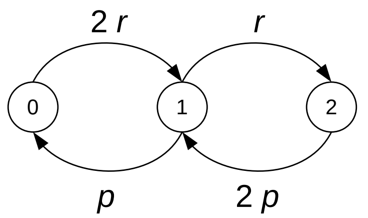
2. **Analysis of Markov chains in continuous time** **(22 P.)**

a) Draw a diagram of states and transitions of an example of a Continuous Time Markov Chain (CTMC) in which some states are transient and all recurrent states communicate! Explain which states are transient! (3 P.)

b) Draw a diagram of states and transitions of an example of a CTMC in which some states are transient and not all recurrent states communicate! Explain which states do not communicate! (3 P.)

c) Which condition must hold so that a finite CTMC possesses a steady-state distribution? What if this condition does not hold? (2 P.)

d) Consider the following diagram of a CTMC:



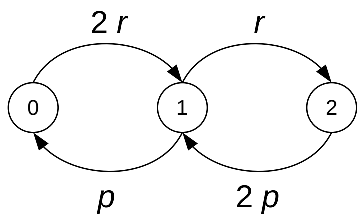
i. Give the generator matrix Q of this CTMC! (3 P.)

ii. Give (in matrix notation!) the balance equation(s) required to determine the vector of steady-state probabilities π. For any ergodic CTMC, this system of equation always possesses a particular property. Which is it? (2 P.)

iii. Give the required normalization constraint for this CTMC! (1 P.)

iv. Define first appropriate vectors or matrices and then write down (in matrix notation!) equations that permit us to determine steady state probabilities for the CTMC depicted above! (4 P.)

v. Assume that the CTMC depicted below describes a workstation with two parallel servers that can operate, fail, and get repaired independently.



Assume that the workstation is never starved nor blocked, that each of the two servers fails with rate p and gets repaired with rate r. Then the states of the system represent the number of operational servers. Assume furthermore that each server (if it is operational) processes workpieces with rate μ. Assume that you have determined the vector π of steady-state probabilities. Now give and explain briefly a formula to determine the average throughput of the system! (4 P.)

3. **Modeling of two-machine lines via Continuous Time Markov Chains (CTMCs) (11 P.)**

Consider a two-machine line with the following features:

- The first machine is never starved, the second machine is never blocked.
- Between the two machines there is a buffer for workpieces with limited capacity C.
- Both machines are reliable.
- Processing times at both machines are exponentially distributed with rate $\mu_i, i = 1, 2$.
- The yield at the first machine is uncertain: With probability $\frac{1}{3}$ the yield is one workpiece, with probability $\frac{2}{3}$ it is two workpieces.
- The first machines operates according to a Blocking-before-Service protocol: It only starts its process if there is sufficient space in the buffer so that all workpieces resulting from the first machine's process can be placed in the buffer. This means that we can never have a situation in which a workpiece already processed on the first machine rests on this machine because the buffer is full.
- The second machine operates in a parallel batch mode: It can process two workpieces at a time. However, if only a single workpiece should be available when the second machine becomes free again, it processes this single workpiece.

Denote with $s = s(n)$ the state of the system. Here n with $0 \leq n \leq N$ is the number of workpieces in the system that have already been processed by the first machine and have not yet left the system. We hence have an extended buffer size $N = C + 1$.

a) Assuming $C = 4$, draw the diagram of states and transitions! (5 P.)

b) Assume that you have determined the steady-state probabilities π_n of being in system state n. Now give and explain formulas to determine the following quantities:

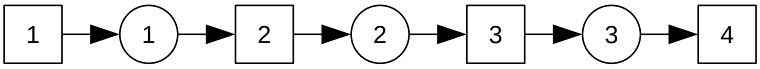
i. Throughput PR_1 via the first machine: (2 P.)

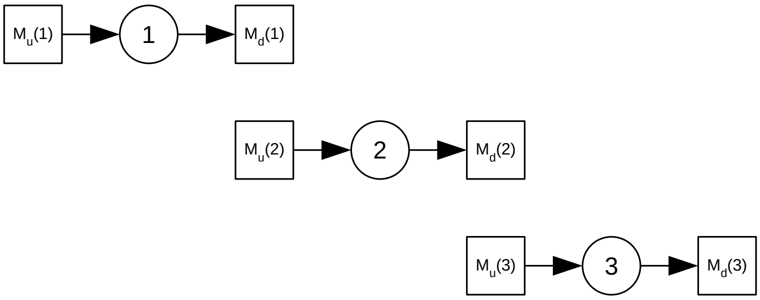
ii. Throughput PR_2 via the second machine: (2 P.)

iii. Average inventory $\overline{n}$: (2 P.)

4. **Decomposition of unreliable flow lines** **(9 P.)**

Consider the model of a Markovian flow line with limited buffer capacities and exponentially distributed processing time, times to failure and repair times with rates $\mu_i, p_i, r_i, i = 1, ..., I$ which we studied over the course of the semester. Assume that it is analyzed via the two-machine decomposition approach depicted below for the example of a four-machine line.





a) Give and explain the general definition of the virtual upstream machine of virtual line i being up, i.e., $\{\alpha_u(i,t) = 1\}$! (3 P.)

b) Using this general definition, apply it to virtual line 1, i.e., give the situation(s) in which the virtual upstream machine of that virtual line is up. (1 P.)

c) Using this general definition, apply it to virtual line 2, i.e., give the situation(s) in which the virtual upstream machine of that virtual line is up. Make use of the recursive nature of the definition of virtual machine states to eliminate all references to virtual machines upstream of the virtual upstream machine of line 2. (2 P.)

d) Using this general definition, apply it to virtual line 3, i.e., give the situation(s) in which the virtual upstream machine of that virtual line is up. Make use of the recursive nature of the definition of virtual machine states to eliminate all references to virtual machines upstream of the virtual upstream machine of line 3. (3 P.)