Leibniz Universität Hannover
Wirtschaftswissenschaftliche Fakultät
Institut für Produktionswirtschaft
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Exam
on
**Manufacturing Systems Modeling and Analysis**
Winter Term 2021-2022

**Hints:**

1. The exam consists of **10** pages (including this front page). Please check that your copy is complete and complain immediately if it is not.
2. Answer all questions and solve all given problems.
3. You are given 60 minutes to work on the exam and you can score a total of 60 points.
4. You may answer the questions using either the German or the English language.
5. **Show your work!** If you use a formula to solve a problem, present it in its general form first!
6. You may use a single double-sided and hand-written help sheet in letter format or DIN-A4 format with any content you may find helpful to work on the exam.
7. You may use a pocket calculator.
8. For this exam written in a university computer lab you may use the desktop computer provided by the university and the Matlab files related to this exam that have been provided in class and stored locally on this machine for your usage while working on the exam.

**Personal data:**

| Family name | Given name | Matriculation number | Study program |
|---|---|---|---|
| | | | |

**Rating:**

| Task | 1 | 2 | 3 | Sum |
|---|---|---|---|---|
| Score | | | | |

1. **The exponential and the Poisson distribution** **(21 P.)**

   Assume that service times $T$ at a machine are exponentially distributed with service rate $\mu = \frac{1}{4}\mathrm{h}^{-1}$.

   a) Give for this random variable $T$ both general formulas and the concrete numerical values (including appropriate units) of

      i. the expected value, (2 P.)

      ii. the standard deviation, (2 P.)

      iii. and the coefficient of variation. (2 P.)

   b) Determine the probability Prob$(1\ \mathrm{h} \leq T \leq 2\ \mathrm{h})$! (3 P.)

c) Determine the conditional probability Prob$(T \geq 2\ \mathrm{h} | T \geq 1\ \mathrm{h})$! (2 P.)

d) Determine the conditional probability Prob$(T \geq 1\ \mathrm{h} | T \geq 4\ \mathrm{h})$! (2 P.)

e) Assume that the machine is operating continuously. Determine the probability of observing three service completions during a time interval of length 10 hours. (2 P.)

f) Show formally that the exponential distribution possesses the memorylessness property! Explain each step of your derivation! (6 P.))

2. **Modeling of two-machine lines via Continuous Time Markov Chains (CTMCs) (14 P.)**

   Consider a two-machine line with the following features:

   - The first machine is never starved, the second machine is never blocked.
   - Between the two machines there is a buffer for workpieces with limited capacity $C$.
   - The first machine is unreliable. While it is operating, it fails with rate $p_1$, i.e., we have operation-dependent failures. After a failure, i.e., when it is down, it gets repaired with rate $r_1$.
   - The second machine is reliable.
   - Processing times at both machines are exponentially distributed with rate $\mu_i, i = 1, 2$.
   - The operation at the first machine is a batch process: Each operation completed on the first machine always yields two work pieces.
   - The second machine always processes one work piece at a time.
   - The first machines operates according to a Blocking-before-Service protocol: It only starts its process if there is sufficient space in the buffer so that both work pieces resulting from the first machine's process can be placed in the buffer. This means that we can never have a situation in which a work piece already processed on the first machine rests on this machine because the buffer is full.

   Denote with $s = s(n, \alpha_1)$ the state of the system. Here $n$ with $0 \leq n \leq N$ is the number of work pieces in the system that have already been processed by the first machine and have not yet left the system. We hence have an extended buffer size $N = C + 1$. If the first machine is operational, we have $\alpha_1 = 1$, if that machine is down, we have $\alpha_1 = 0$.

   a) Assuming $C = 5$, draw the diagram of states and transitions and show which states are transient! (8 P.)

b) Assume that you have determined the steady-state probabilities $\pi_{n,\alpha_1}$ of being in system state $s(n, \alpha_1)$. Now give and explain formulas to determine the following quantities:

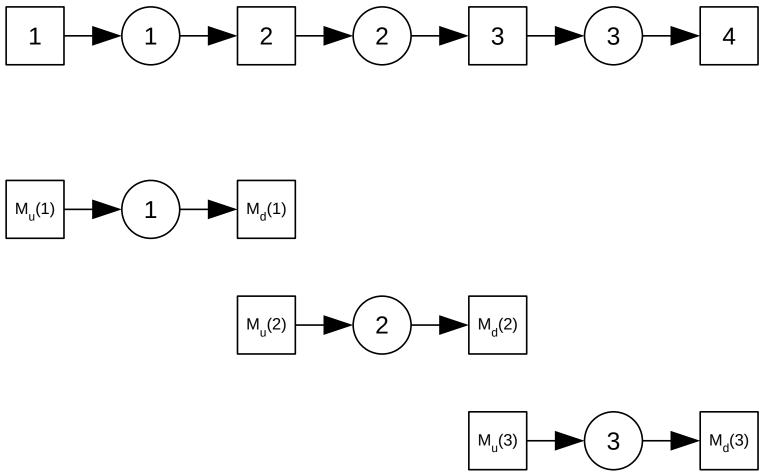
i. Throughput $PR_1$ via the first machine: (2 P.)

ii. Throughput $PR_2$ via the second machine: (2 P.)

iii. Average inventory $\overline{n}$: (2 P.)

3. **Decomposition of unreliable flow lines** **(25 P.)**

Consider the model of a Markovian flow line with limited buffer capacities and exponentially distributed processing time, times to failure and repair times with rates $\mu_i, p_i, r_i, i = 1, ..., I$ which we studied over the course of the semester. Assume that it is analyzed via the two-machine decomposition approach depicted below for the example of a four-machine line.




a) Consider a virtual line $i$ of the decomposition with a virtual downstream machine $M_d(i)$ with rates $\mu_d(i), p_d(i)$, and $r_d(i)$. Give and explain a Flow-Rate-Idle-Time equation for that virtual line to determine the throughput of that virtual line from the perspective of the downstream machine $M_d(i)$. (5 P.)

b) Now consider machine $i$ in the original line. Using the parameters of that machine and assuming that you have the information about probabilities of blocking and/or starving of that machine $i$, express the throughput $PR_i$ of that machine in terms of those quantities to determine the Flow-Rate-Idle-Time equation for that machine $i$ of the original line. Is this an exact or only approximate relationship? Explain! (5 P.)

c) Give and explain the general definition of the downstream machine of virtual line $i$ being up, i.e., $\{\alpha_d(i,t) = 1\}$! (3 P.)

d) Give and explain the general definition of the virtual downstream machine of virtual line $i$ being down, i.e., $\{\alpha_d(i,t) = 0\}$! (3 P.)

e) Using this general definition of a virtual downstream machine being up (see task 3c), apply it to virtual line 3 of the four-machine-line example introduced above, i.e., give the situation(s) in which the virtual downstream machine of that virtual line 3 is up. (1 P.)

f) Using this general definition of a virtual downstream machine being up, apply it to virtual line 2, i.e., give the situation(s) in which the virtual downstream machine of that virtual line is up. Make use of the recursive nature of the definition of virtual machine states to eliminate all references to virtual machines downstream of the virtual downstream machine of line 2. (2 P.)

g) Show the initial steps of a derivation to determine the repair rate $r_d(i)$ of that virtual downstream machine by developing an expression for the repair probability $r_d(i)\delta t$ during a short time interval of length $\delta t$. Use the method of decomposition on the conditioning event to develop an expression of the form $r_d(i)\delta t = A \cdot B + C \cdot D$ with $A, B, C$, and $D$ as conditional probabilities for which you have to develop a first expression (6 P.)