Leibniz Universität Hannover
Wirtschaftswissenschaftliche Fakultät
Institut für Produktionswirtschaft
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Exam
on
Manufacturing Systems Modeling and Analysis
Winter Term 2022-2023

Hints:

1. The exam consists of **13** pages (including this front page). Please check that your copy is complete and complain immediately if it is not.
2. Answer all questions and solve all given problems.
3. You are given 60 minutes to work on the exam and you can score a total of 60 points.
4. You may answer the questions using either the German or the English language.
5. **Show your work!** If you use a formula to solve a problem, present it in its general form first!
6. You may use a single double-sided and hand-written help sheet in letter format or DIN-A4 format with any content you may find helpful to work on the exam.
7. You may use a pocket calculator.

Personal data:

Family name	Given name	Matriculation number	Study program

Rating:

Task	1	2	3	4	Sum
Score					

1. **Poisson process, the exponential and the Poisson distribution** **(15 P.)**

   Assume that inter-arrival times T at a machine are exponentially distributed with arrival rate $\lambda = \frac{1}{5}\mathrm{h}^{-1}$.

   a) Give for this random variable T the general formulas as well as the concrete numerical values (including appropriate units) of

      i. the expected value, (2 P.)

      ii. the standard deviation, (2 P.)

      iii. and the coefficient of variation. (1 P.)

   b) Determine the probability Prob$(1\ \mathrm{h} \leq T \leq 4\ \mathrm{h})$! (2 P.)

c) Determine the conditional probability $\text{Prob}(T \leq 4\ \text{h} | T \geq 1\ \text{h})$! (2 P.)

d) Determine the conditional probability $\text{Prob}(T \geq 4\ \text{h} | T \leq 1\ \text{h})$! (1 P.)

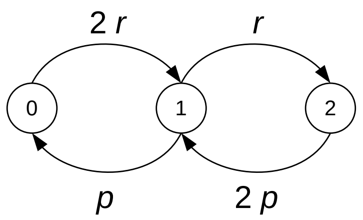
e) Determine the conditional probability $\text{Prob}(T \geq 1\ \text{h} | T \geq 4\ \text{h})$! (1 P.)

f) Determine the probability of observing *at least* three arrivals during a time interval of length 20 hours. (3 P.)

g) In a Poisson arrival process, what is the distribution describing the arrival times? (1 P.))

2. **Analysis of Markov chains in continuous time** **(10 P.)**

Consider the following diagram of a CTMC:



a) Give the generator matrix Q of this CTMC! (3 P.)

b) Give (in matrix notation!) the balance equation(s) required to determine the vector of steady-state probabilities π. For any ergodic CTMC, this system of equation always possesses a particular property. Which is it? (2 P.)

c) Give the required normalization constraint for this CTMC! (1 P.)

d) Define first appropriate vectors or matrices and then write down (in matrix notation!) equations that permit us to determine steady state probabilities for the CTMC depicted above! (4 P.)

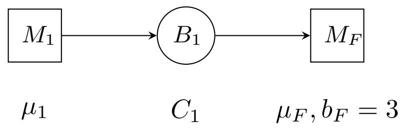
3. **Modeling of flow lines via Continuous Time Markov Chains (CTMCs)** **(17 P.)**

The following tasks related to the analysis of flow lines with limited buffer capacities consist of two parts. Solving the first part first will help you massively to deal with the second.

Always assume in a steady-state system analysis that

- the capacities of buffers holding intermediate products are limited,
- processing times at machines follow exponential distributions,
- machines are reliable, i.e., they never fail,
- upstream machines are never starved,
- upstream machines operate according to a blocking-after-service protocol, and
- downstream machines are never blocked.

a) Consider first a two-machine line as depicted below.



The machine M_1 upstream of the buffer B_1 operates with rate μ_1. It processes workpieces one-by-one, i.e., it always operates on *single* workpieces (as we typically assumed in class). The buffer B_1 behind machine M_1 can hold C_1 intermediate workpieces. The downstream machine M_F making the final product, however, operates in a batch mode with batch size $b_F = 3$. It only starts its operation if it has been loaded with $b_F = 3$ intermediate workpieces. The processing rate of machine M_F is μ_F. Upon completion of the process on machine M_F, a single unit of the final product made from the entire batch of size $b_F = 3$ of intermediate products leaves the system. Hence the maximum number of intermediate workpieces still in the system that have already been processed by machine M_1 is $N_1 = 1 + C_1 + 3 = C_1 + 4$.

Denote with $s = s(n_1)$ the state of the system. Here n_1 with $0 \le n_1 \le N_1$ is the number of work pieces of the intermediate product in the system that have already been processed by machine M_1, but not yet by machine M_F (and hence have not yet left the system).

i. Assuming $C_1 = 3$, draw the diagram of states and transitions and show which states are transient, if any! (6 P.)

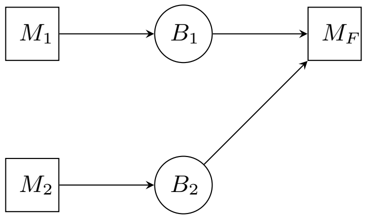
ii. Assume that you have determined the steady-state probabilities π_n of being in system state $s(n)$. Now give and explain formulas to determine the following quantities:

   A. Throughput TH_1 *in terms of final products* via the first machine: (2 P.)

   B. Throughput TH_2 *in terms of final products* via the second machine: (2 P.)

   C. Average inventory $\overline{n}$ *in terms of final products* : (2 P.)

iii. Now assume that two machines M_1 and M_2 with processing rates μ_1 and μ_2, each with a separate downstream buffer B_1 and B_2, respectively, make two different types of intermediate products, which are then assembled in a batch process by machine M_F operating with a processing rate μ_F. The system structure is shown below.

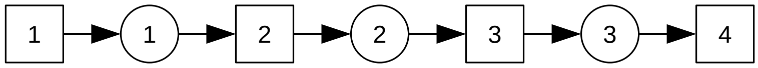


This batch process requires $b_{F,1}$ units of the type-1 intermediate products stemming from machine M_1 *and* further $b_{F,2}$ units of the type-2 intermediate product stemming from machine M_2, in order to create a single unit of the final product.

Assuming buffer sizes $C_1 = C_2 = 0$ and batch sizes $b_{F,1} = b_{F,2} = 2$, draw the diagram of states and transitions! (5 P.)

4. **Structural behavior of flow lines** **(18 P.)**

Consider the model of a Markovian flow line with I machines with limited buffer capacities $C_i, i = 1, ..., I-1$ and exponentially distributed processing times, times to failure and repair times with rates $\mu_i, p_i, r_i, i = 1, ..., I$ which we studied over the course of the semester. Assume that the first machine is never starved, the last never blocked, we have blocking after service and operation-dependent failures. Below you find the system structure for the example of a four-machine line, i.e., $I = 4$.



Unless explicitly stated otherwise, assume that all machines have the *same* processing rates μ, failure rates p, and repair rates r. Likewise, assume that all buffers are of equal size C.

Consider the case of a four-machine line. On the following pages, draw graphs and give explanations about the system behavior as you systematically vary selected system parameters.

a) Assume that we vary the processing rate μ_2 of the second machine from 0 to $3 \cdot \mu$, while leaving all other system parameters unchanged.

   i. Draw and explain the structural behavior of the throughput TH of the entire line over μ_2. (3 P.)

   ii. Draw and explain the structural behavior of the average inventory level $\overline{n}_2$ of the buffer behind the second machine over μ_2. (3 P.)

b) Assume that we increase the failure rate p_2 of the second machine starting at $p_2 = 0$, while leaving all other system parameters unchanged.

   i. Draw and explain the structural behavior of the throughput TH of the entire line over p_2. (3 P.)

   ii. Draw and explain the structural behavior of the average inventory level $\overline{n}_2$ of the buffer behind the second machine over p_2. (3 P.)

c) Assume that we increase the repair rate r_2 of the second machine starting at $r_2 = 0$, while leaving all other system parameters unchanged.

   i. Draw and explain the structural behavior of the throughput TH of the entire line over r_2. (3 P.)

   ii. Draw and explain the structural behavior of the average inventory level $\overline{n}_2$ of the buffer behind the second machine over r_2. (3 P.)