Leibniz Universität Hannover
Wirtschaftswissenschaftliche Fakultät
Institut für Produktionswirtschaft
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Exam
on
Stochastic Models in Production and Logistics
Summer Term 2019

Hints:

1. The exam consists of **10** pages (including this front page). Please check that your copy is complete and complain immediately if it is not.
2. Answer all questions and solve all given problems.
3. You are given 60 minutes to work on the exam and you can score a total of 60 points.
4. You may answer the questions using either the German or the English language.
5. **Show and explain your work!** If you use a formula to solve a problem, presents it in its general form first!
6. You may use a single double-sided and hand-written help sheet in in letter format or DIN-A4 format with any content you may find helpful to work on the exam.
7. You may use a pocket calculator.

Personal data:

Family name	Given name	Matriculation number	Study program

Rating:

Task	1	2	3	Sum
Score				

1. **Poisson arrival processes** **(10 P.)**

   Assume a stationary Poisson arrival process to a service system with arrival rate $\lambda = \frac{1}{2}\text{h}^{-1}$. Let T denote the interarrival times.

   a) Determine the standard deviation of the random variable T. (2 P.)

   b) Determine the probability $\text{Prob}(2\ \text{h} \leq T \leq 8\ \text{h})$. (3 P.)

c) Determine the conditional probability Prob$(T \geq 4 \text{ h}|T \geq 1 \text{ h})$. (2 P.)

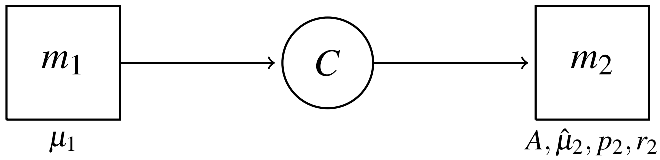
d) Determine the conditional probability Prob$(T \geq 1 \text{ h}|T \geq 4 \text{ h})$.) (2 P.)

e) Determine the probability to have 4 arrivals during the next six hours, given that 2 arrivals occurred during the last three hours! (2 P.)

2. **Steady-state analysis of the $M/M/1$ queueing system** **(10 P.)**

   Explain the assumptions of the $M/M/1$ queueing model and show how to derive closed-form expressions for the steady-state probabilities of having $n \geq 0$ customers in the system!

3. **Analysis of a Markovian two-stage flow line with limited buffer capacities** **(40 P.)**



<u>Hint:</u> Read the following text slowly, carefully, and repeatedly! Take your time!

Consider the two-machine flow line depicted above. Assume that the first machine is never starved and the second never blocked. Assume that the buffer between the machines can hold up to C parts.

For the first machine, assume *blocking after service* (BAS). The processing times at the first machine follow an exponential distribution with rate μ_1. This first machine is reliable, i.e., it never fails.

The processing times at the second machine also follow an exponential distribution. The parameter of that exponential distribution depends on the state of the second machine.

Inside of the second machine, up to A functional units of a critical component work simultaneously on the current workpiece unless this second machine is starved. The average processing rate of a *single* unit of that critical component at the second machine is $\hat{\mu}_2$.

Due to the simultaneous operation of the different units of the critical component of that machine on the (at most one) workpiece processed at any moment in time, the instantaneous rate of that exponential processing time distribution is *proportional* to the current number $\alpha = 1, ..., A$ of functional units of the critical component. The second machine is therefore operational if at least one unit of the critical component is operational.

Assume that the different units of the critical component used inside of the second machine can only fail while the second machine is working (*operation dependent failures*, (ODF)). Assume furthermore, that they can fail independently.

Both the times to failure and the repair times are exponentially distributed. An *individual* unit of the critical component fails with rate p_2 while the second machine is working. A failed unit gets repaired with rate r_2, independently of the state of the rest of the system.

The state space of the system is described as a tuple $s = s(n, \alpha_2)$ such that n denotes the number of parts in the system that have already been processed by the first machine whereas α_2 denotes the number of functional units of the critical component at the second machine.

a) Assume that the buffer can hold $C = 2$ work pieces and that the second machine has $A = 3$ units of the critical component. Draw the diagram of all states and transitions and classify the states. (16 P.)

b) Does this system possess a steady-state solution for system state probabilities? Explain your answer! (3 P.)

c) Write down the general balance equation for the steady-state probability of the state $s(2,1)$. (3 P.)

d) Write down two different formulae to determine the throughput TH of the system via the first and the second machine, respectively. (4 P.)

e) Write down a formula to determine the average inventory level $\overline{n}$ of the system. (2 P.)

f) Consider the entire random response time R that a workpiece spends in the system from the moment that it enters the first machine to the moment that it leaves the second machine. Give and explain a formula to determine the expected value $\mathrm{E}[R]$ of that time. (Hint: Ask yourself how this expected value was computed for the $M/M/1$ queueing model and apply that train of thought in an analogous matter by using your previous results!) (3 P.)

g) Sketch and explain a graph to show the qualitative nature of the impact of the buffer size C on the throughput TH of the line! (3 P.)

h) Sketch and explain a graph to show the qualitative nature of the number A of units of the critical component on the throughput TH of the line! (3 P.)

i) Sketch and explain a graph to show the qualitative nature of the number A of units of the critical component on the average inventory level $\bar{n}$ of the line! (3 P.)